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Bohm criterion for a plasma composed of electrons and positive dust grains

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An investigation is presented of a collisionless sheath of a dusty plasma whose constituents are positively charged dust grains and electrons. Accounting for the Boltzmann electron density distribution and the hydrodynamic dust fluid model, supplemented by Poisson and dust charging equations, a space-charge sheath model is developed. The Bohm criterion for a dusty plasma system with electrons and positive dust grains is deduced. The results can have relevance to space and laboratory plasmas.

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I. INTRODUCTION

During the last decade, there has been a great deal of interest [1–4] in investigating numerous collective processes in a dusty plasma. Collective processes involve studies of the dust grain shielding [5,6], the dusty plasma sheath [7,8], and the excitation of waves and instabilities [9–11] as well as coherent nonlinear structures [12–14]. The simplest coherent nonlinear structure, which can exist in a bounded plasma without the dust grain, is a steady-state collisionless space-charge sheath at a wall or a negative electrode. It is, therefore, of significant interest to investigate an effect which can be produced on the sheath by the presence of charged dust particles. From the point of view of applications, such a problem is of great importance in connection with the determination of the dust flux to walls or electrodes in laboratory glow discharges, and in material processing as well as in dusty space plasmas.

If a plasma consists of positive ions, electrons, and dust particles of a constant charge, the problem of a space-charge sheath is essentially the same as that in a plasma with multiple ion species, which has been studied both in the case of various positive ion species [15–17], and in the case of positive and negative ion species (e.g., Refs. [18–20], and references therein). However, charges on the dust grain surface may vary owing to the variation of the electron and ion currents that reach the dust grain surface during the dust grain charging. The account of dust charge fluctuations in the dust acoustic and dust ion-acoustic waves results in a number of interesting effects [21–25]. One can expect that the variability of a dust particle charge will also result in some interesting effects [8] in the theory of the near-wall space-charge sheath.

The simplest model of a dusty plasma is the one accounting for atoms, electrons, and positive dust particles. We note that dust grains can acquire positive charge due to thermionic emission [26] and photoemission [27–29]; in a thermal dusty plasma, the dust grains are mostly charged positively [30]. The theory of dust acoustic waves in such a plasma, with account of the variable charge of the dust particles, was developed in Ref. [31]. In this paper, we formulate a model for a space-charge sheath in an electron-positive dust plasma, and present a Bohm criterion (e.g., Refs. [32–34], and references therein).

The paper is organized in the following fashion. In Sec. II, we present our steady-state sheath model based on the Boltzmann electron distribution and the hydrodynamic equations for positively charged dust grains; the governing equations are supplemented by Poisson's and dust charging equations. The Bohm criterion is developed in Sec. III, where numerical results are also presented. Section IV contains concluding remarks and possible applications.

II. MODEL

We consider a stationary space-charge sheath at a planar surface in an unmagnetized dusty plasma whose constituents are atoms, electrons, and positively charged dust particulates. The surface is under a potential which is about or below the floating potential, i.e., the density of the electron current to the surface is of the same order of magnitude of or smaller than the density of current delivered by dust grains. It is assumed that at large distances from the surface (at the "sheath edge"), where the plasma is quasineutral, plasma parameters tend to constant values, which will be designated with the index s.

One should expect that the velocity of the directed motion of dust particles in the direction towards the surface is in the sheath of the order of $\sqrt{Z_s T_e / m_d}$, where T_e is the electron temperature in energy units and Z and m_d are the charge number and the mass of a dust particle, respectively. It follows that the density of the electric current transported by the particles is of the order of $eZ_s n_{ds} \sqrt{Z_s T_e / m_d}$, where e is the magnitude of the electron charge and n_d is the number density of the dust particles. Since the density of the net electric current does not change across the sheath, and since at the surface it is of the same order as the density of the current transported by the dust particles, the density of the net electric current in the sheath is of the order of $eZ_s n_{ds} \sqrt{Z_s T_e / m_d}$, and this is also the order of the net electron current density.

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The density of the electron thermal current is of the order of $en_{es}\overline{C_e}$, where n_e is the electron number density (note that $n_{es}=Z_s n_{ds}$ in the approximation considered), $\overline{C_e}=\sqrt{8T_e/\pi m_e}$ is the mean electron thermal velocity, and m_e is the electron mass. It follows that the ratio of the net electron current in the sheath to the electron thermal current is of the order of $\sqrt{Z_s m_e/m_d}$. Since $Z_s \ll m_d/m_e$, the above ratio is small and the distribution of the electron number density in the sheath may be described by the Boltzmann distribution, namely,

$$n_e = n_{es}\exp(e\phi/T_e), \tag{1}$$

where ϕ is the electrostatic potential (for definiteness, zero of the potential is chosen at the sheath edge).

We assume that the temperature of the gas of dust particles is much smaller than the electron temperature. Then the energy of the directed motion of dust particles toward the surface substantially exceeds the energy of their thermal motion, and all the dust particles at a given point of the sheath have approximately the same velocity, which is directed toward the surface. Assuming that the motion of dust particles is unaffected by impacts of electrons and atoms, one can write, the continuity and momentum equations for the dust particles as

$$n_d v_d = J_d, \quad m_d v_d \frac{dv_d}{dy} = -eZ\frac{d\phi}{dy}, \tag{2}$$

respectively, where the y axis is directed from the surface into the plasma, v_d is the dust particle velocity in the direction toward the surface, and J_d is an unknown constant (the number density of the dust flux to the surface).

Assume that the time of flight of a dust particle across the sheath substantially exceeds the mean time between successive electron impacts suffered by the particle or the mean time between successive emission acts. Note that this assumption implies, generally speaking, that the variation of the charge number Z across the sheath is much larger than unity. Then charging of a dust particle may be considered as a continuous process, and may be described by the equation

$$-v_d\frac{dZ}{dy} = \frac{I^+ - I^-}{e}, \tag{3}$$

where I^+ is the emission current and I^- is the electron collection current. Expressions for these currents are written in the forms

$$I^+ = eAf, \quad I^- = eB\frac{n_e}{n_{es}}g, \tag{4}$$

where A and B are constants; they are associated with characteristic frequencies of the emission acts and of the impact of the plasma electrons suffered by a dust particle, respectively, $f=f(Z)$ and $g=g(Z)$ are dimensionless functions of (a local value of) Z. For definiteness, we assume that the normalization factors A and B are defined in such a way that $f(Z_s)=1$, $g(Z_s)=1$. For example, if dust is charged by UV irradiation [27], then

$$A = \pi R^2 JY\exp(-\sigma\gamma_s), \quad f = \exp[-\sigma(\gamma-\gamma_s)], \tag{5}$$

where R is the radius of a spherical dust particle, J is the flux density of the UV photon beam, Y is the yield of photoelectrons per one photon, $\gamma = Ze^2/RT_e \equiv e\varphi/T_e$, φ is the surface potential of the dust grain, $\gamma_s = Z_s e^2/RT_e$, $\sigma = T_e/T_p$, and T_p is the average energy of the emitted photoelectrons. Similar expressions can also be written for the case when dust is charged by thermionic emission [26].

Assuming that the dust grain size is much smaller than the electron Debye radius, one can employ for the electron collection current an expression given by the orbital-limited motion theory; see, e.g., Refs. [35,36]. Then one can write

$$B = (1+\gamma_s)\pi R^2 n_{es}\overline{C_e}, \quad g = \frac{1+\gamma}{1+\gamma_s}. \tag{6}$$

We note that the assumption of a constant value Z_s of the dust particle charge number at the sheath edge implies that Z_s is a root of the equation $A=B$.

We assume that the intergrain spacings are much smaller than the sheath thickness. (Since the latter is of the order of the electron Debye radius, this amounts to assuming that the number of grains in the Debye sphere is large.) Then the dust charge distribution in space may be considered as continuous, and the Poisson equation reads

$$\frac{d^2\phi}{dy^2} = 4\pi e(n_e - Zn_d). \tag{7}$$

We introduce dimensionless variables

$$\eta = \frac{y}{\lambda_D}, \quad V = \frac{v_d}{v_{ds}}, \quad \tilde{Z} = \frac{Z}{Z_s}, \quad \Phi = \frac{e\phi}{T_e}, \tag{8}$$

where $\lambda_D = \sqrt{T_e/4\pi n_{es}e^2}$ is the electron Debye radius, and $v_{ds}=J_d/n_{ds}$ is the dust particle velocity at the sheath edge. The system of equations now assumes the forms

$$VV' = -(1-\beta)\tilde{Z}\Phi', \quad V\tilde{Z}' = \alpha(ge^{\Phi} - f),$$
$$\Phi'' = e^{\Phi} - \tilde{Z}/V, \tag{9}$$

where prime denotes differentiation with respect to η, and

$$\alpha = \frac{4\lambda_D}{v_{ds}Z_s}, \quad \beta = 1 - \frac{Z_s T_e}{m_d v_{ds}^2}. \tag{10}$$

Obviously, the parameter α, as introduced here, represents the ratio of the scale of the sheath thickness, λ_D, to the length scale of variation of the charge of a dust particle.

In order to deduce an initial-value problem, it is convenient to introduce new independent variable $\tilde{\Phi} = -\Phi$. Equations (9) then assume the forms

$$V\frac{dV}{d\tilde{\Phi}}=(1-\beta)\tilde{Z},\quad EV\frac{d\tilde{Z}}{d\tilde{\Phi}}=\alpha(f-ge^{-\tilde{\Phi}}),$$

$$E\frac{dE}{d\tilde{\Phi}}=\frac{\tilde{Z}}{V}-e^{-\tilde{\Phi}}, \tag{11}$$

where $E=\Phi'$ is the dimensionless electric field. These equations should be solved at the interval $[0,eU/T_e]$, where U is the (absolute value of the) sheath voltage, with initial conditions

$$V(0)=1,\quad \tilde{Z}(0)=1,\quad E(0)=0. \tag{12}$$

III. BOHM CRITERION

We seek a solution at small $\tilde{\Phi}$ in the forms

$$V=1+P\tilde{\Phi}^p+\cdots,\quad \tilde{Z}=1+Q\tilde{\Phi}^q+\cdots,\quad E=W\tilde{\Phi}^w+\cdots, \tag{13}$$

where p, P, q, Q, w, and W are (real) constants to be determined later. For expansion (13) to be consistent with boundary conditions (12), p, q, and w should be positive. Furthermore, it should be $w\geqslant 1$, since the integral

$$\eta=\int_{\tilde{\Phi}}^{eU/T_e} d\tilde{\Phi}/E \tag{14}$$

must diverge as $\tilde{\Phi}\to 0$ (which corresponds to $\eta\to\infty$).

Expanding the first equation in Eqs. (11), one finds that $p=1$ and $P=1-\beta$. Expanding the other equations, one arrives at

$$WQq\tilde{\Phi}^{w+q-1}=\alpha(\tilde{\Phi}-\Gamma Q\tilde{\Phi}^q) \tag{15}$$

and

$$W^2w\tilde{\Phi}^{2w-1}=\beta\tilde{\Phi}+Q\tilde{\Phi}^q, \tag{16}$$

where

$$\Gamma=\left.\frac{d(g-f)}{d\tilde{Z}}\right|_{\tilde{Z}=1}. \tag{17}$$

We note that the electron collection current increases with an increase of the potential of the surface of a dust particle while the emission current decreases due to enhanced reflection by the potential barrier. It follows that $\Gamma>0$.

Assuming that all the terms of Eqs. (15) and (16) are of the same order, one should set $w=q=1$. Equation (15) gives

$$Q=\frac{\alpha}{W+\alpha\Gamma}. \tag{18}$$

Substituting this relationship into Eq. (16), one arrives at a cubic equation for W:

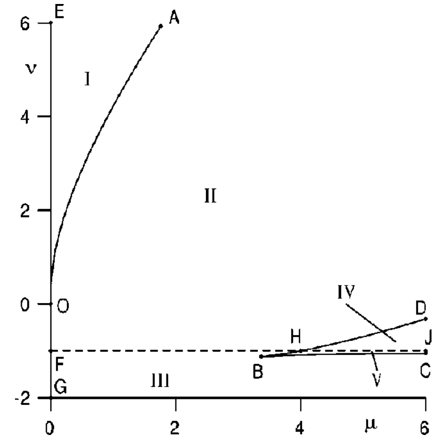

FIG. 1. Roots of the discriminant of Eq. (20). OA: $\nu_1=\nu_1(\mu)$, Eq. (A1), and the first equation in Eq. (A3). BC: $\nu_2=\nu_2(\mu)$, the second equation in Eq. (A3). BD: $\nu_3=\nu_3(\mu)$, Eq. (A4).

$$W^3+\alpha\Gamma W^2-\beta W-\alpha(1+\beta\Gamma)=0. \tag{19}$$

The above equation may be converted to a two-parameter form by means of the substitution $W=\Gamma^{-1/2}z$,

$$z^3+\sqrt{\mu}z^2-\nu z-\sqrt{\mu}(1+\nu)=0, \tag{20}$$

where $\mu=\alpha^2\Gamma^3$ and $\nu=\beta\Gamma$. The discriminant of Eq. (20) is

$$D=4\nu^3-8\mu\nu^2+4\mu(\mu-9)\nu+4\mu^2-27\mu. \tag{21}$$

The roots of the cubic polynomial on the right-hand side of Eq. (21) are given by Eqs. (A1), (A3), and (A4) of the Appendix, and are depicted in Fig. 1. Equation (20) has three real roots in the domains $\{\mu\geqslant 0,\nu\geqslant\nu_1\}$ and $\{\mu\geqslant 27/8,\nu_2\leqslant\nu\leqslant\nu_3\}$. These roots are given by Eqs. (A9) and (A10) of the Appendix. In the rest of the half-plane $\{\mu\geqslant 0,-\infty<\nu<\infty\}$, Eq. (20) has a real and two complex roots.

Assuming that the electric field in the vicinity of the sheath edge is directed toward the sheath, one should look for a positive root of Eq. (20). It follows that the considered problem is well posed at such points of the half-plane $\{\mu\geqslant 0,-\infty<\nu<\infty\}$ for which Eq. (20) has just one positive root, other roots being negative or complex.

One can see that Eq. (20) has a zero root on the lines $\mu=0$ and $\nu=-1$ [lines $EOFG$ and FHJ in Fig. 1; point H here is defined by the equation $\nu_3(\mu)=-1$ and has coordinates (4, -1)]. In the vicinity of these lines, the roots in question are

$$z=-\left(1+\frac{1}{\nu}\right)\sqrt{\mu}+\cdots,\quad z=\sqrt{\mu}(\nu+1)+\cdots, \tag{22}$$

respectively. It follows that this root is positive in the (right) vicinity of OF and in the upper vicinity of FHJ; the root is negative in the vicinity of OE, in the vicinity of FG, and in the lower vicinity of FHJ.

Other roots of Eq. (20) on lines $EOFG$ and FHJ take values

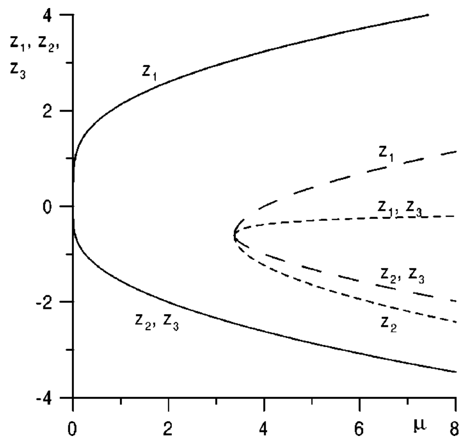

FIG. 2. Values of roots of Eq. (20) on lines *OA*, *BC*, and *BD*. Solid line: $z_i = z_i[\mu, \nu_1(\mu)]$ ($i=1,2,3$); line *OA*. Short dashes: $z_i = z_i[\mu, \nu_2(\mu)]$; *BC*. Long dashes: $z_i = z_i[\mu, \nu_3(\mu)]$; *BD*.

$$z = \pm\sqrt{\nu}, \quad z = \frac{-\sqrt{\mu} \pm \sqrt{\mu - 4}}{2}, \tag{23}$$

respectively. It follows that Eq. (20) has, apart from the zero root, a positive root and a negative root on *OE*, two complex roots on *OG* and *FH*, and two negative roots on *HJ*.

The behavior of the roots on lines *OA*, *BC*, and *BD* is shown in Fig. 2. One can see that Eq. (20) has a positive root and two (equal) negative roots on *OA*; three negative roots on *BC* and *BH*; a positive root and two negative roots on *HD*.

Consider now domains *I*-*V* shown in Fig. 1. (The domains do not include boundaries.) One can conclude that Eq. (20) has a positive root and two negative roots in domain *I*, a positive root and two complex roots in domain *II*, a negative root and two complex roots in domain *III*, a positive root and two negative roots in domain *IV*, and three negative roots in domain *V*.

It follows that the problem is well-posed in the domains *I*, *II*, and *IV*, i.e., at

$$\nu > -1. \tag{24}$$

A condition of well posedness of a sheath problem is usually referred to as the Bohm criterion; see, e.g., Refs. [32–34]. Thus inequality (24) represents the Bohm criterion for the problem considered. Another form of this inequality is

$$v_{ds} > \left(\frac{\Gamma Z_s T_e}{(\Gamma+1) m_d} \right)^{1/2}, \tag{25}$$

IV. CONCLUDING DISCUSSION

The Bohm criterion appears in the above treatment as a condition of well posedness of the sheath problem transformed to the independent variable $\tilde{\Phi}$: under this condition, a solution exists with the electric field at the sheath edge, $\tilde{\Phi} = 0$, directed toward the surface (and such a solution is unique). On the other hand, the Bohm criterion may be interpreted as a condition of well posedness of the sheath problem in the original statement. Indeed, one can check readily that Eq. (19) multiplied by W coincides with the characteristic equation at which one arrives while seeking an exponential solution (with the exponential factor written as $e^{-W\eta}$) to Eq. (9) linearized around the boundary condition at $\eta \to \infty$. One can see from Fig. 2 that on each of the lines *OA*, *BC*, and *BD* equal roots are negative. Since Eq. (20) has no imaginary roots at $\mu > 0$, one can conclude that real parts of complex roots in the domains *II* and *III* are negative. It follows that inequality (24) represents a condition under which nontrivial solutions to Eq. (9) exist which are compatible with the boundary conditions at the sheath edge (and these solutions are monotonic at the sheath edge and belong to a one-parameter family).

Inequality (24) [or (25)] does not involve α. In other words, the effect of the variability of charge of dust particles manifests itself only through parameter Γ.

We note that once inequality (24) is independent of α, it can be derived in a simpler way, by means of considering the limiting case $\alpha \to \infty$. A solution to Eq. (19) in this limit is $W_1 = -\alpha\Gamma, W_{2,3} = \pm\sqrt{\beta + \Gamma^{-1}}$. Thus Eq. (19) has a positive root provided that $\beta > -\Gamma^{-1}$, in accord with Eq. (24).

The above analysis does not apply in cases when one or more roots of Eq. (19) are zero. Indeed, $W=0$ means that the respective solution at small $\tilde{\Phi}$ cannot be sought in the form of Eq. (13) with $w=1$. It follows that the above analysis does not apply on line *FHJ*, and that is why the Bohm criterion appears in the above analysis in the form of a strict inequality. On the other hand, it is well known that the Bohm criterion usually allows the equality sign and, furthermore, is usually valid just in the marginal form. Therefore, one can expect that the right-hand side of inequality (25) gives the value of the dust particle velocity at the sheath edge. It can be seen easily that this quantity coincides with the velocity of low frequency ($\omega \ll A/Z_0$), long wavelength (in comparison with the electron Debye radius) dust acoustic waves in collisionless dust plasmas with a variable dust particle charge and a low translational temperature of the dust.

In the limiting case $\alpha = 0$, the model of a sheath in a dusty plasma with a variable dust particle charge, as formulated in Sec. II, becomes identical to the model of a sheath in a plasma with positive particles of a constant charge (or with multiply charged ions). Therefore, the Bohm criterion in the limiting case $\alpha = 0$ is given by the same formula as that of the conventional Bohm criterion in a plasma with multiply charged ions, viz.

$$v_{ds} \geqslant \left(\frac{Z_s T_e}{m_d} \right)^{1/2}, \tag{26}$$

One can see that Eq. (25), which does not involve α, does not provide a limiting transition in the case $\alpha \to 0$. On the other hand, Eq. (25) conforms to Eq. (26) in another limiting case, namely, $\Gamma \to \infty$. [The same result may be obtained by imposing the limit $\Gamma \to \infty$ directly on Eq. (19): a solution in this limit is $W_1 = -\alpha\Gamma, W_{2,3} = \pm\sqrt{\beta}$, and there is just one positive root provided that β is positive.] Note that this conclusion is consistent with Eq. (18): when $\Gamma \to \infty$, Q goes to zero and the particle charge at the sheath edge is constant. The physical sense of this result may be understood as follows. The charging of a particle at the sheath edge is caused by the variations of the emission current and of the electron collection current (the unperturbed values of these currents are equal and do not cause any charging). The latter variations are caused by variations of the local potential and of the particle charge. If the dependence of the emission current and of the electron collection current on the particle charge is strong ($\Gamma \gg 1$), then the variation of the particle charge must be small; otherwise it would cause a variation of the currents which cannot be compensated for either by a variation of the currents caused by a variation of potential or by accumulation of the charges on the surface of a particle.

The results of our investigation should be used, in particular, for evaluating the dust flux towards a wall or toward a negative electrode, or for the calculation of the characteristic of an electrostatic probe in a dusty plasma which is composed of atoms, electrons and positively charged dust grains.

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APPENDIX: SOLUTION OF EQ. (20)

If $\mu < 27/8$, the discriminant of Eq. (20) [which is given by the cubic polynomial on the right-hand side of Eq. (21)] has one real root

$$\nu_1 = -\Delta_1 \operatorname{cosec} 2\delta_1 + \frac{2\mu}{3}, \tag{A1}$$

where

$$\Delta_1 = \frac{2}{3}\sqrt{\mu^2 + 27\mu}, \quad \delta_1 = \arctan\left(\tan\frac{\delta_2}{2}\right)^{1/3},$$
$$\delta_2 = \arcsin\frac{8\sqrt{\mu(\mu+27)^3}}{8\mu^2 - 540\mu - 729}. \tag{A2}$$

If $\mu > 27/8$, the discriminant has three real roots

$$\nu_1 = \Delta_1 \cos\delta_3 + \frac{2\mu}{3}, \quad \nu_2 = -\Delta_1 \cos\left(\delta_3 - \frac{\pi}{3}\right) + \frac{2\mu}{3}, \tag{A3}$$

$$\nu_3 = -\Delta_1 \cos\left(\delta_3 + \frac{\pi}{3}\right) + \frac{2\mu}{3}, \tag{A4}$$

where

$$\delta_3 = \frac{1}{3}\arccos\frac{-8\mu^2 + 540\mu + 729}{8\sqrt{\mu(\mu+27)^3}}. \tag{A5}$$

Note that

$$\mu \to 0: \quad \nu_1 = \frac{3}{2^{2/3}}\mu^{1/3} + \cdots, \tag{A6}$$

$$\mu = \frac{27}{8}: \quad \nu_1 = 9, \quad \nu_2 = -\frac{9}{8}, \quad \nu_3 = -\frac{9}{8}, \tag{A7}$$

$$\mu \to \infty: \quad \nu_1 = \mu + \cdots, \quad \nu_2 = -1 + \cdots, \quad \nu_3 = \mu + \cdots. \tag{A8}$$

The real roots of Eq. (20) may be written as

$$z_1 = \Delta_2 \cos\delta_4 - \frac{\sqrt{\mu}}{3}, \quad z_2 = -\Delta_2 \cos\left(\delta_4 - \frac{\pi}{3}\right) - \frac{\sqrt{\mu}}{3}, \tag{A9}$$

$$z_3 = -\Delta_2 \cos\left(\delta_4 + \frac{\pi}{3}\right) - \frac{\sqrt{\mu}}{3}, \tag{A10}$$

where

$$\Delta_2 = \frac{2}{3}\sqrt{\mu + 3\nu}, \quad \delta_4 = \frac{1}{3}\arccos\frac{\sqrt{\mu}(-2\mu + 18\nu + 27)}{2(\mu + 3\nu)^{3/2}}. \tag{A11}$$

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